Statistical limits on isotropic CR distributions with a space detector

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Outline

- Introduction
- Simulation details
- Methods for sky map production and analysis
- Results for the isotropic sky
- Results for simulated anisotropic skies
- Conclusions

Introduction

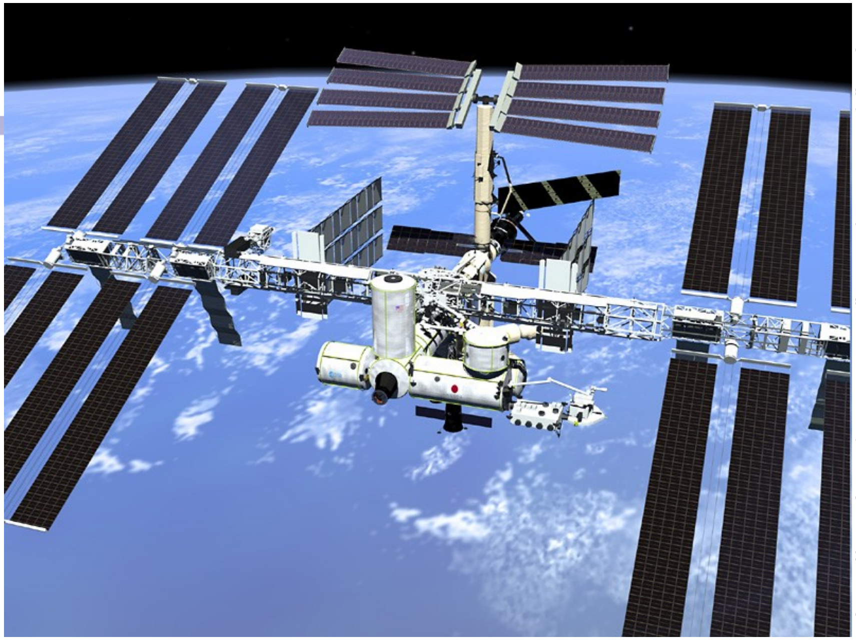
- The study of CR isotropy can result crucial in, e. g.:
  - probing the propagation mechanism in the heliosphere and in the galaxy;
  - searching of possible sources.

- Because of tiny anisotropic signals ~$10^{-3}/10^{-4}$, detection of anisotropy over the full sky requires:
  - high statistics;
  - a refined method of analysis of sky maps.

➡ Our method is based on the spherical harmonics expansion of the sky map of the ratio between the observed sky and the expected isotropic sky.

➡ The present method can be applied to any kind of CR particle.

Simulation details



http://nssdc.gsfc.nasa.gov/image/spacecraft/iss.jpg

- Positions and timing of the International Space Station (ISS) (orbit inclination 51.6° relative to the Equator, height ~400Km) for a period of ~2.5y.

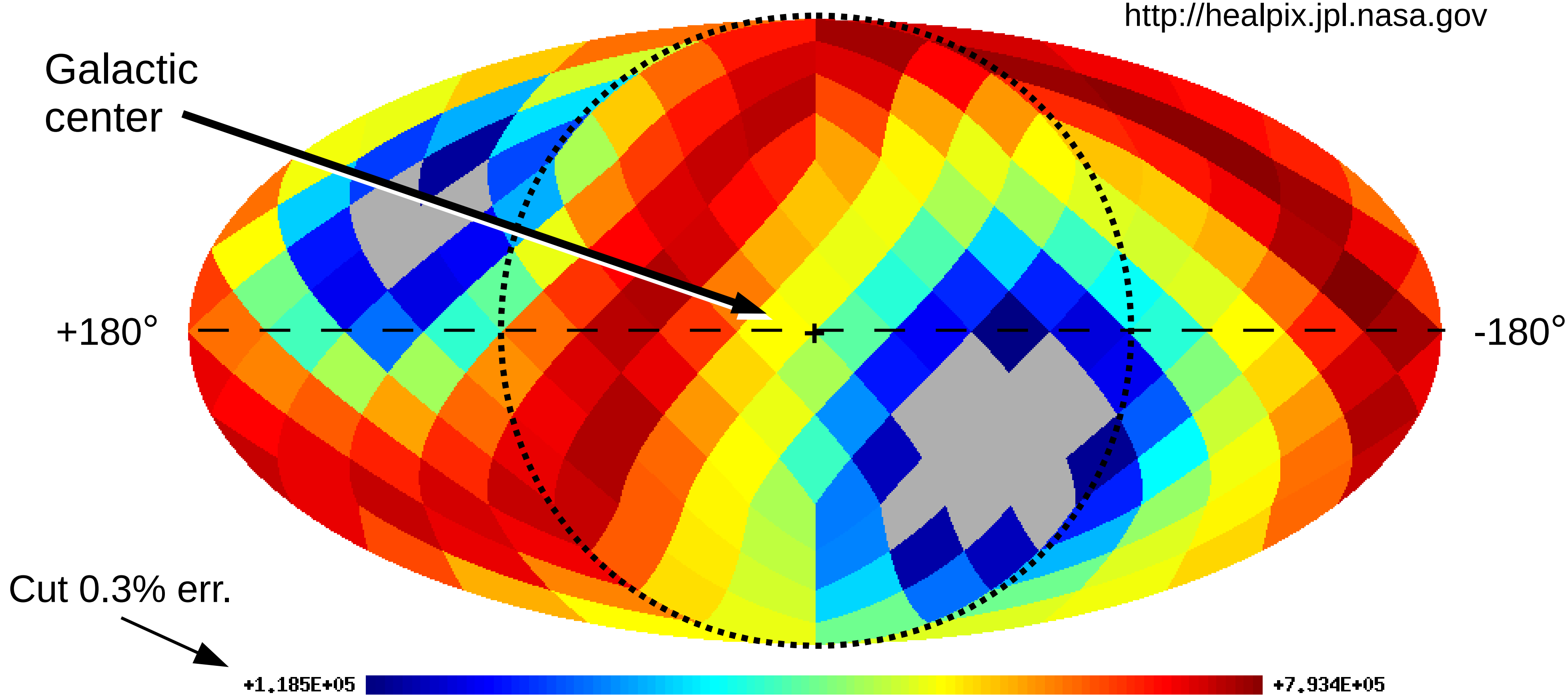
- **~100M positive charged particles (protons)** uniformly generated in a ~40° acceptance cone, distributed in rigidity with a power law ~$R^{-2.78}$ in the rigidity bin [30:100]GV.

- Accounted for 12° tilted zenith relative to ISS reference frame (similar to AMS-02 [M. Aguilar et al. (AMS Collaboration) Phys. Rev. Lett. 110, 141102]), not included any effect due to the detector performance.

Sky pixelization

Simulated protons in galactic coordinates

Celestial sphere pixelization from HEALPix
K.M. Górski,et al., 2005, ApJ 622, 759
http://healpix.jpl.nasa.gov

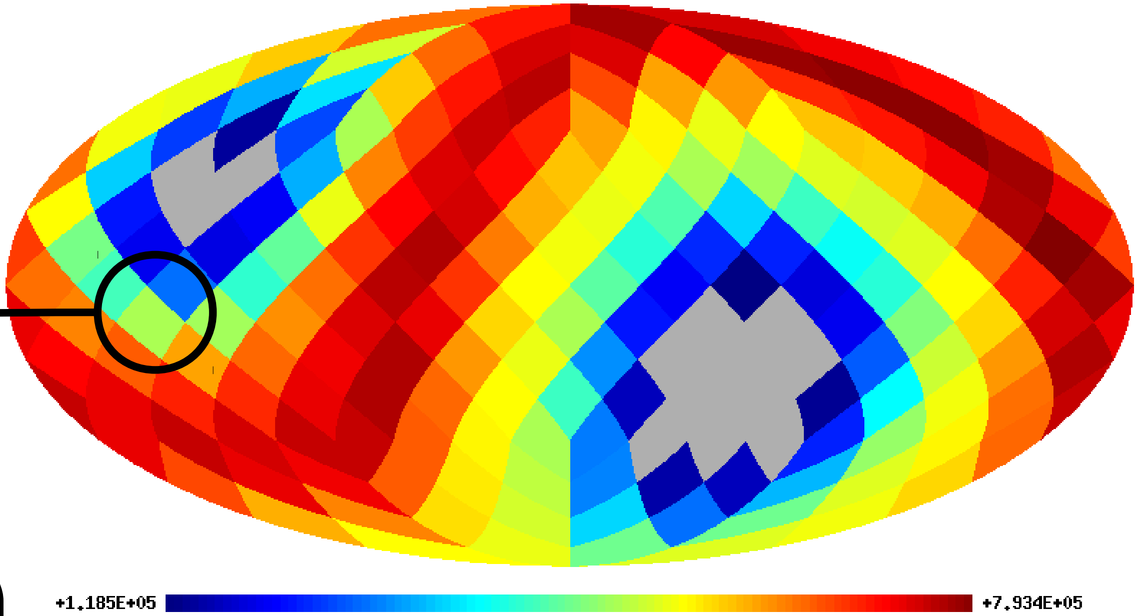


* All pixels have equal areas.
  ► Sampling with **no regional dependence**.

Num. of pixels = 192
Angular pix. size ~14.66°

* Pixels are distributed on iso-latitudinal rings
  on the celestial sphere ► **Regular sampling**.

Methodology



For the i-th pixel, we define:

- Measured proton occupancy level:

$$R_i^{\mathrm{p}} = \frac{\text{num. protons in pixel } i}{\text{total num. protons}}$$

- Generated proton **Isotropic** level:

$$R_i^{\mathrm{gp}} = \frac{\text{num. gen. prot. in pixel } i}{\text{total num. gen. prot.}}$$

Proton level with respect to the isotropic level:

$$\boxed{R_i = \frac{R_i^{\mathrm{p}}}{R_i^{\mathrm{gp}}}}$$

$$\sigma_i^{\mathrm{R}} = \text{error from pixel stat. err.}$$

The reference isotropic sky map

The “**expected isotropic sky map**“ is generated randomly coupling position and time of measure with measured proton directions and rigidities in the apparatus frame [G. L. Cassiday et al., Nucl.Phys. B (Proc. Suppl.) 14A (1990) 291-298]

→ This destroys possible correlations due to external sources, keeping exposure time and total number of events, if all directions are allowed (i.e. above maximum rigidity cut-off).

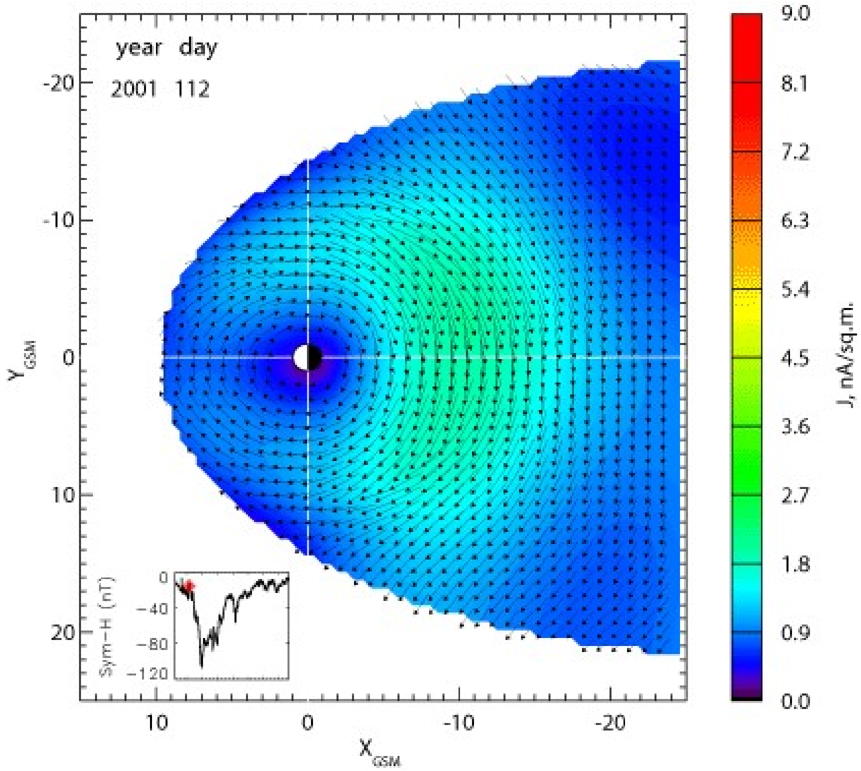
Once the events are generated, their directions are transformed in the specific coordinate system chosen and they constitute the “**isotropic reference**” for the real sky map analysis.

Inside the magnetosphere we generated shuffled events 5x the statistics of the real events.

Back-tracing in magnetosphere

Both generated and shuffled proton trajectories were reconstructed inside the Earth magnetosphere **up to the magnetosphere borders** applying the GeoMag-05 tool [P. Bobik et al. J. of Geophys. Res. v. 111-A5, (2006) http://www.geomagsphere.org/]

It implements the Tsyganenko 2005 [N. Tsyganenko, J. of Geophys. Res. v.110 (2005) A03208] external field model, describing the Earth magnetosphere during **both quiet and active solar periods**.



The analysis is performed with particle asymptotic directions both at the ISS position and at the border of the magnetosphere.

The present analysis was performed on primary particles.

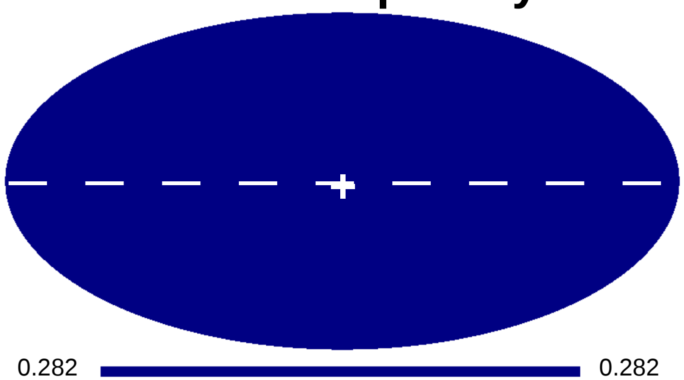
Multipole expansion: monopole, dipole

$$R=\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m} = a_{00} Y_{00} + \boxed{\sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}}$$

ISOTROPIC component

NOT-ISOTROPIC components

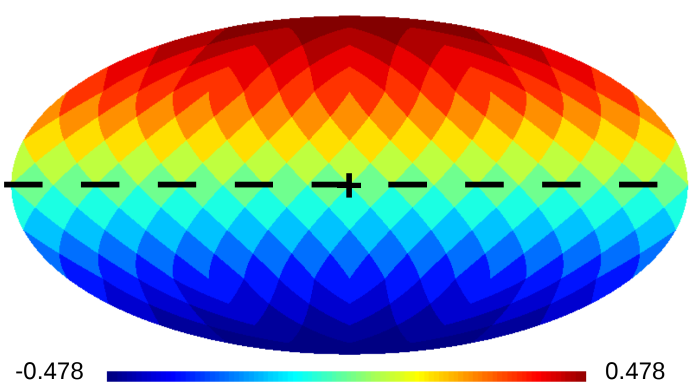
Pure isotropic sky



First not-isotropic component:
DIPOLE $(\ell=1)$

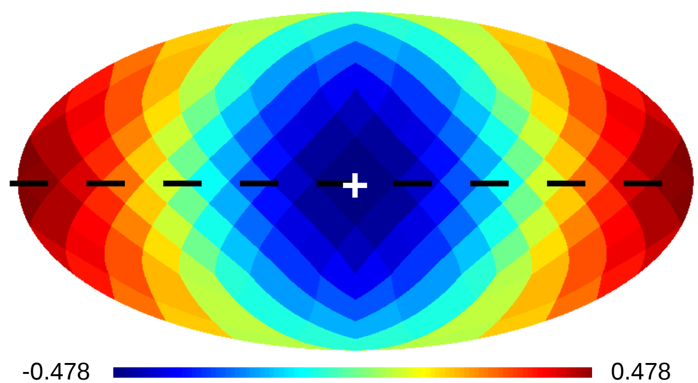
$$\boxed{d_{NS} D_i^{NS} + d_{BF} D_i^{BF} + d_{EW} D_i^{EW}}$$

North-South



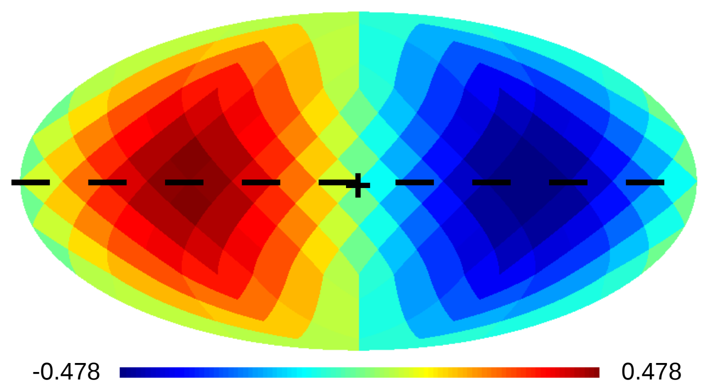
$$\rho_{NS} = \sqrt{\frac{3}{4\pi}} d_{NS}$$

Backward-Forward



$$\rho_{BF} = \sqrt{\frac{3}{4\pi}} d_{BF}$$

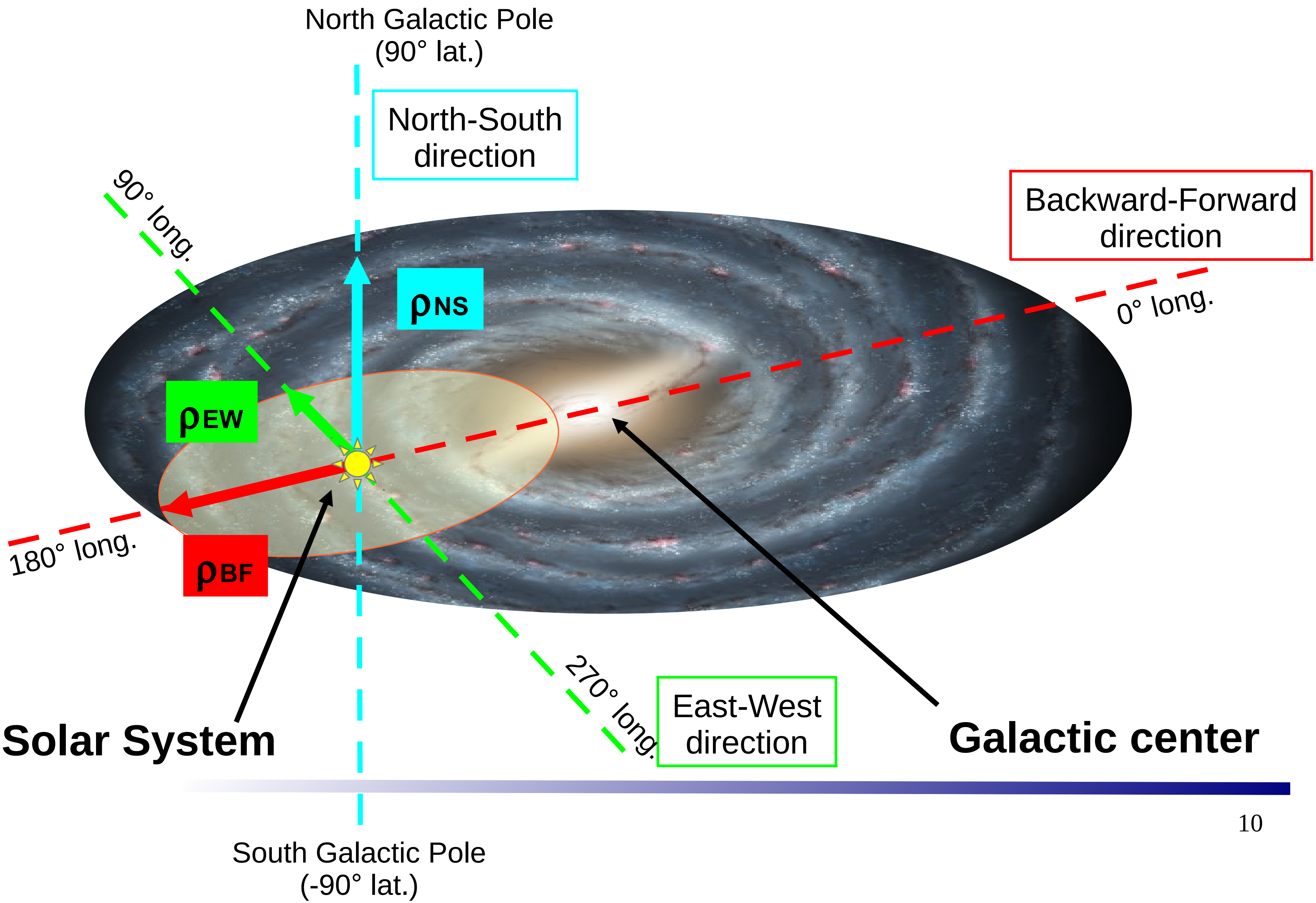
East-West



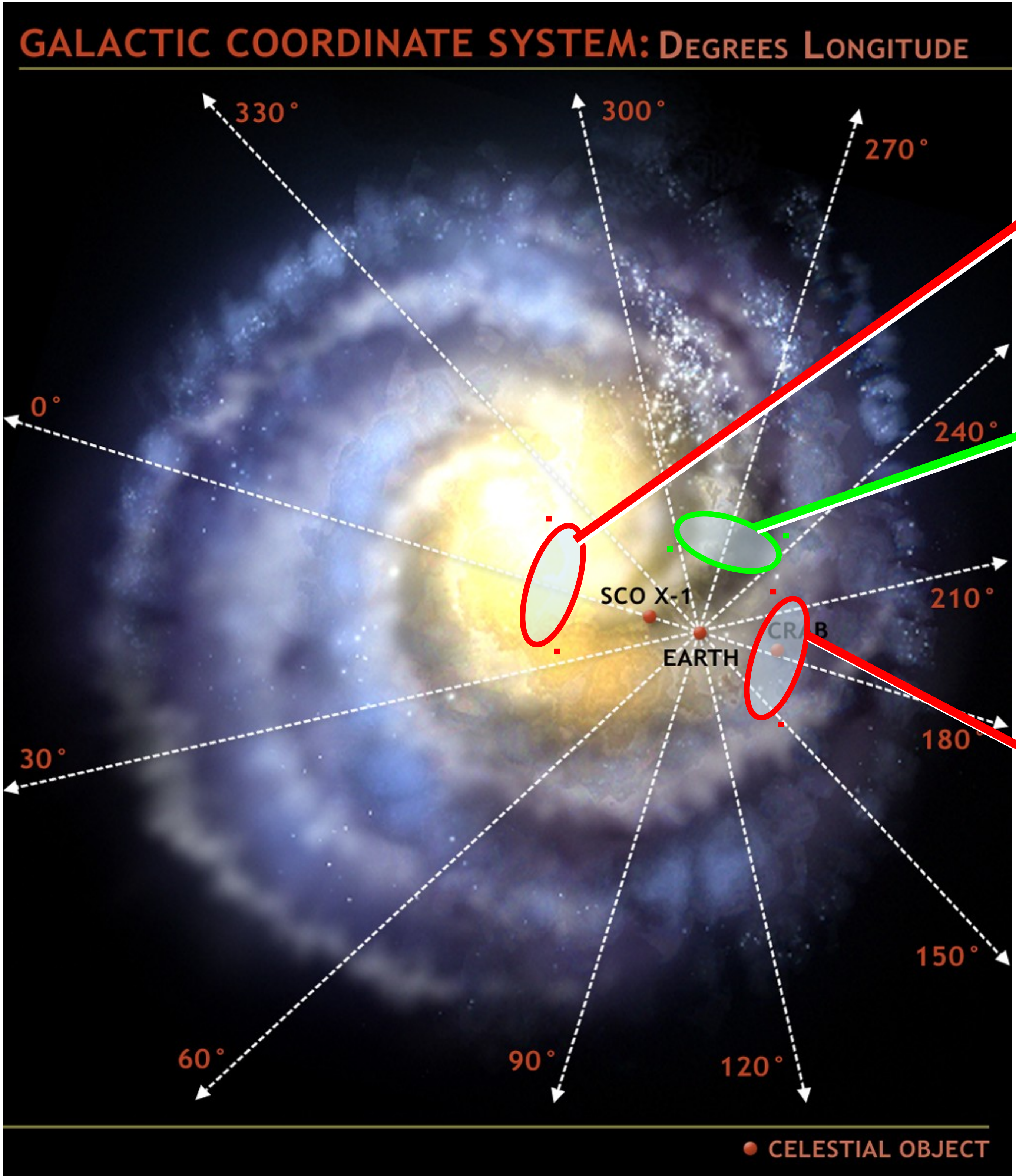
$$\rho_{EW} = \sqrt{\frac{3}{4\pi}} d_{EW}$$

Dipole magnitude: $\delta = \sqrt{\rho_{NS}^2 + \rho_{BF}^2 + \rho_{EW}^2}$

Dipole directions in galactic coordinates



http://chandra.harvard.edu/xray_astro/navigation4.html



Forward region:
- Galactic center, ~8kpc

West region:
- Vela (263°,-3°), 290pc

Backward region:
- Crab (184°,-6°), 2.2kpc
- Geminga (195°,4°), 250pc
- Monogem (201°,8°), 300pc

Fitting the map & error determination

- Multipole expansion parameters:

Chi2 minimization using Minuit:

$$R_i^{calc} = \frac{A_0}{\sqrt{4\pi}} + A_1 D_i^{NS} + A_2 D_i^{BF} + A_3 D_i^{EW} + \dots$$

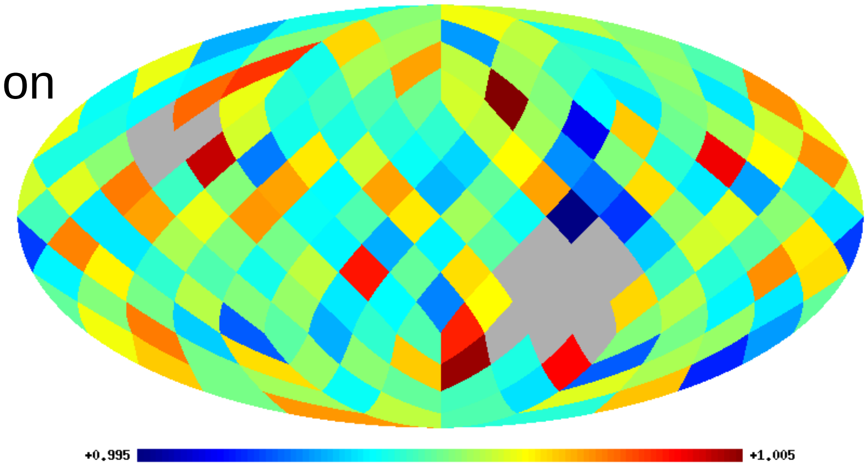
$$\chi^2 = \sum_{i=1}^{N_{pixel}} \left(\frac{R_i - R_i^{calc}}{\sigma_i^R} \right)^2$$

- Error determination:

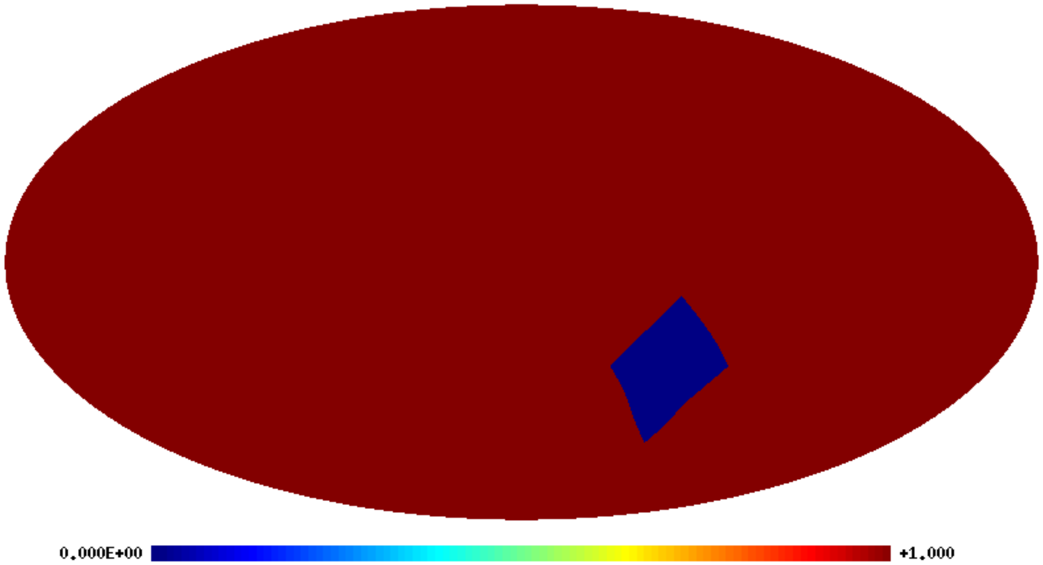
a) **MAP REALIZATION (real measured sky)**
We realize 10,000 maps, randomly choosing pixel value within the pixel error, from the measured proton ratio sky map.

b) **FIT**
the full analysis is performed on each sky realization (i.e. performing a Tminuit fit for each realization).

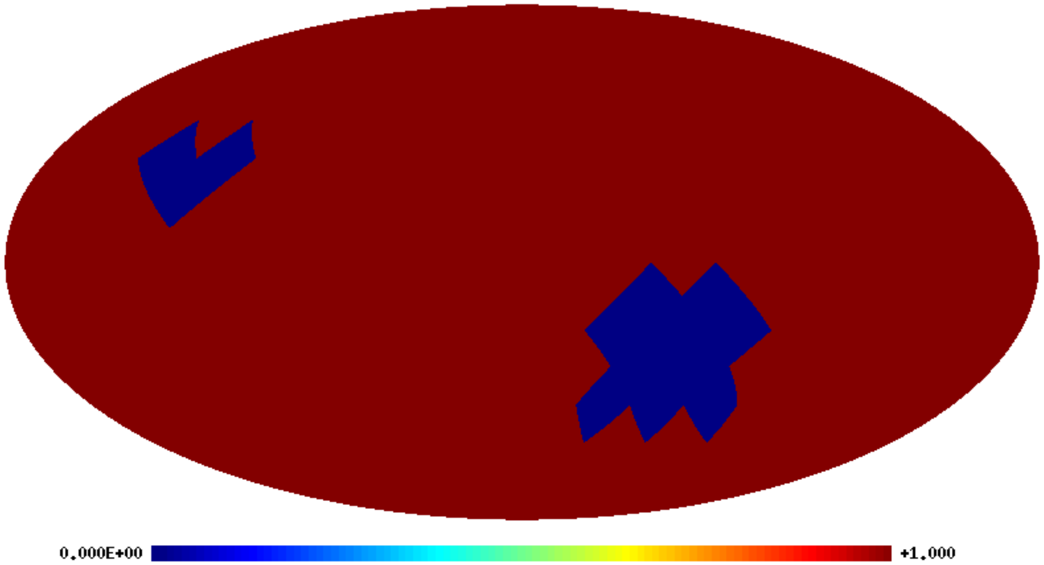
c) **PARAMETER DISTRIBUTIONS**
the estimate of the parameter errors is obtained from the standard deviation of the distribution of the parameters from the fits.



Sky mask



Inside magnetosphere



At magnetosphere border

The holes in the maps correspond to areas around
North and South Geographic Poles,
which are not fully covered by the acceptance cone of the detector.

These pixels are excluded from the fit.

Expected results for MC isotropic sky

Within the present statistics ~100M protons

Statistical limit: we calculate the mean statistical error over active pixels. This value is assigned only to the active pixels themselves.

Only MONOP.		1.0000 ± 0.0001
Only DIPOLE	RhoNS =	0.0000 ± 0.0002
	RhoBF =	0.0000 ± 0.0002
	RhoEW =	0.0000 ± 0.0002

MONOP.		1.0000 ± 0.0001
DIPOLE	RhoNS =	0.0000 ± 0.0002
	RhoBF =	0.0000 ± 0.0002
	RhoEW =	0.0000 ± 0.0002

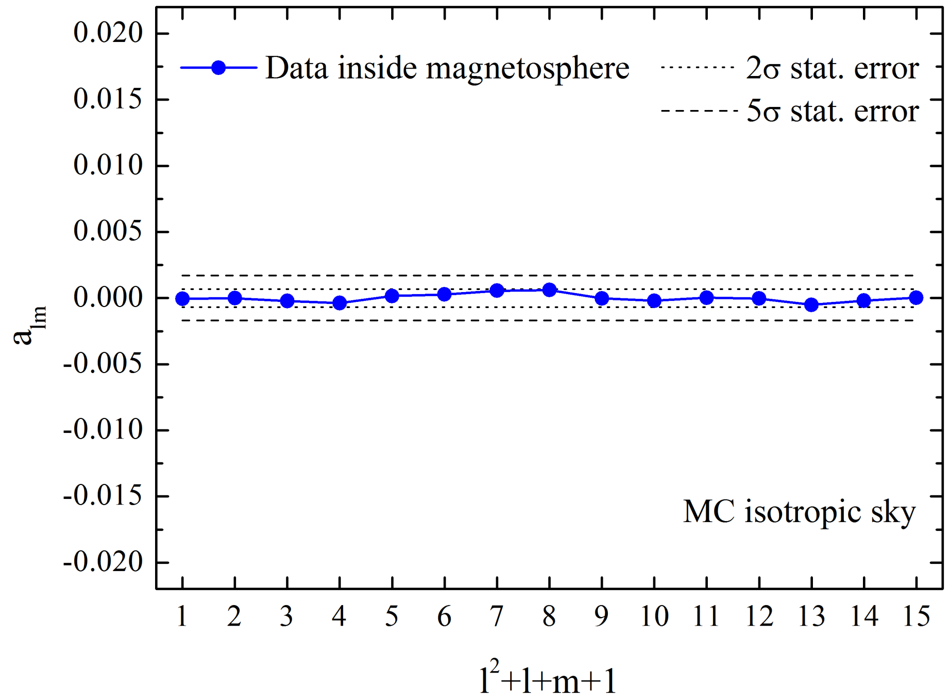
Extrapolated statistical limit for uniformly distributed sky: we calculate the mean statistical error over active pixels. This value is assigned isotropically to each pixel of the map, as if we had a uniform isotropic sky.

Only MONOP.		1.0000 ± 0.0001
Only DIPOLE	RhoNS =	0.0000 ± 0.0002
	RhoBF =	0.0000 ± 0.0002
	RhoEW =	0.0000 ± 0.0002

MONOP.		1.0000 ± 0.0001
DIPOLE	RhoNS =	0.0000 ± 0.0002
	RhoBF =	0.0000 ± 0.0002
	RhoEW =	0.0000 ± 0.0002

In both cases, we exclude dipole strength $\delta > 0.0004$ at 95% CL

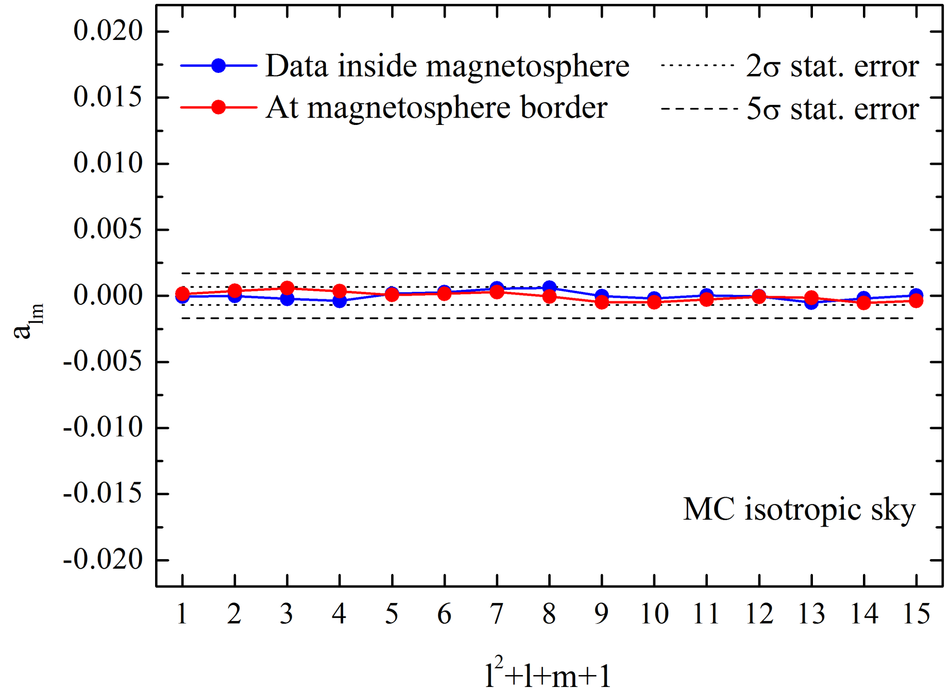
Results for the MC isotropic sky...



INSIDE MAGNETOSPHERE

- The fit has been performed up to the octupole (ℓ=3)
- The a(ℓ,m) coefficients are compatible with the isotropy at ~2σ

...and for the BT isotropic sky



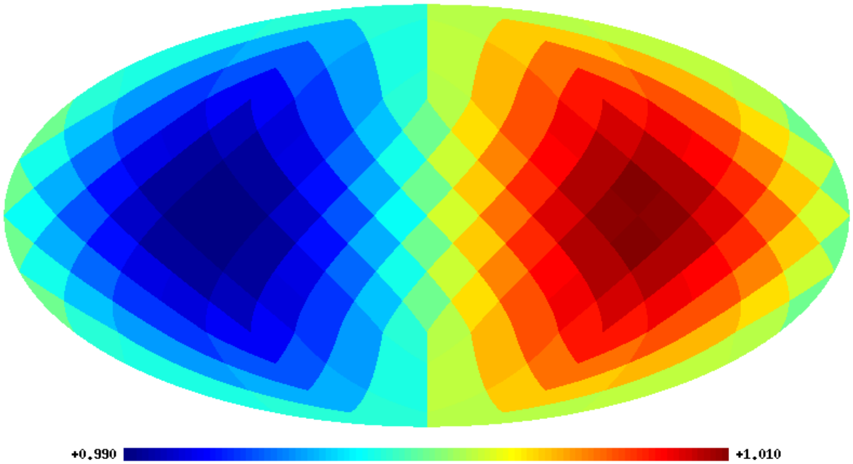
INSIDE MAGNETOSPHERE
and
AT MAGNETOSPHERE BORDER

The a(ℓ,m) coefficients keep compatible with isotropy at ~2σ

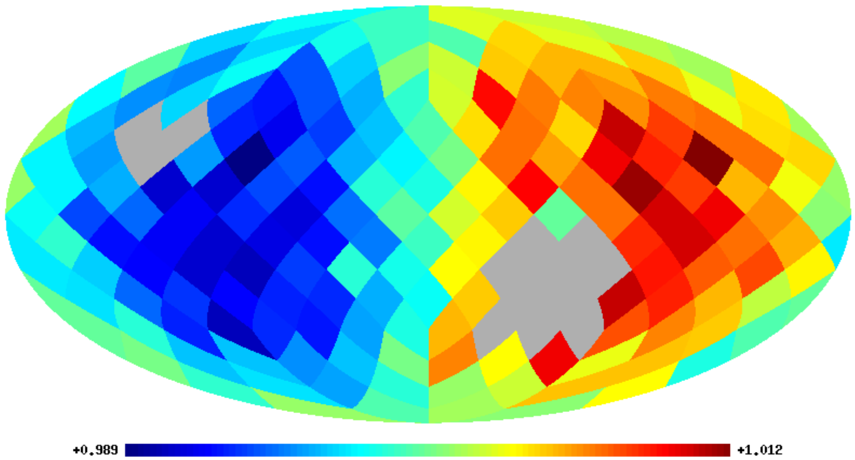
➡ If all directions are allowed, an isotropic distribution of particles keeps its isotropy when traveling through the magnetosphere.

Fermi E. and B. Rossi, Rend. R. Accad. Naz. Lincei **17**, 346 (1933)
M.Vallarta, Encyclopedia of Physics Volume 9 / 46 / 1, 1961, pp 88-129

Simulating the dipole signal



1



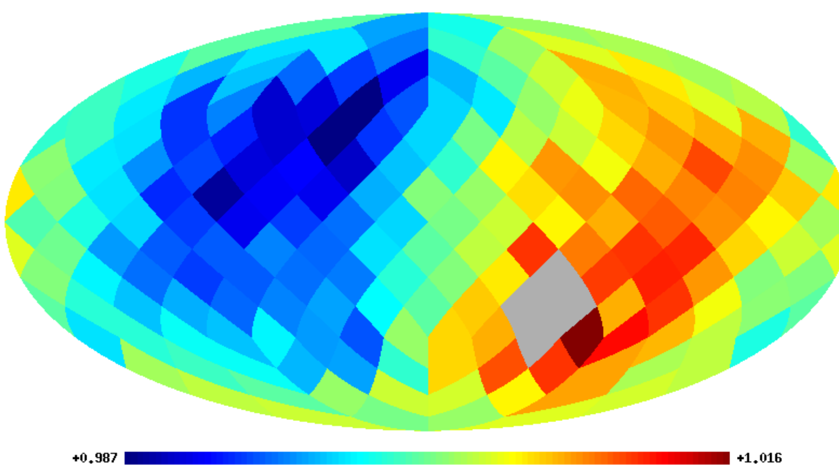
At magnetosphere border

2

1. We simulated a **dipole signal** aligned along the EW direction **at the border of the magnetosphere**, weighting each particle depending on its reconstructed direction in the sky.

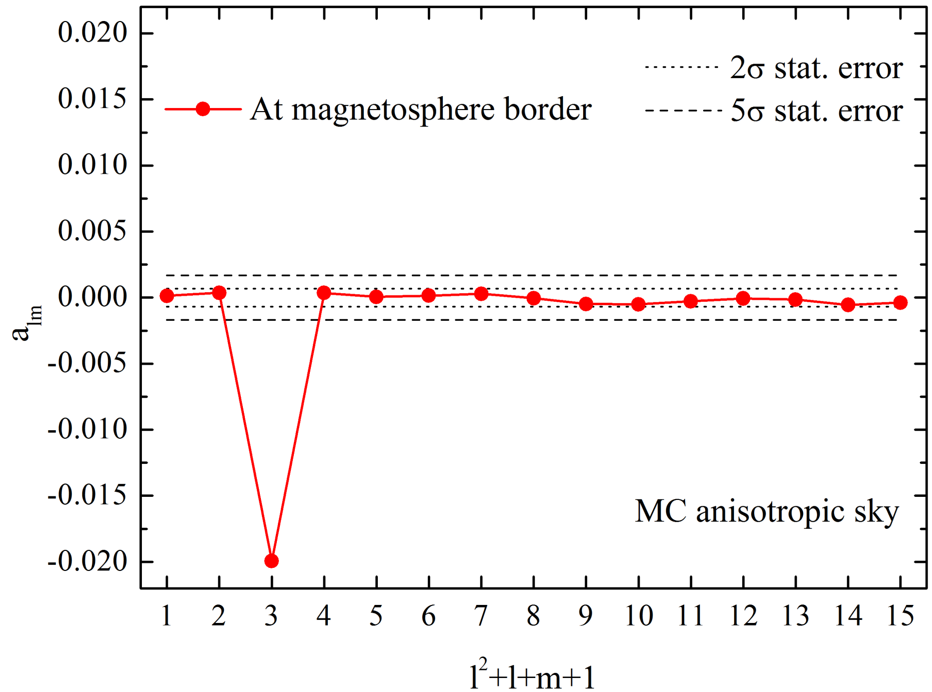
2. Exploiting the one-to-one function given by the BT, we transfered the weight from the magnetosphere borders to the ISS position (i. e., **inside the magnetosphere**).

The procedure can be generalized to any signal intensity and direction.



Inside magnetosphere

Results for the simulated dipole...



AT MAGNETOSPHERE BORDERS

Simulated signal:

$\rho_{NS}=0$

$\rho_{BF}=0$

$\rho_{EW}=-0.01$

i.e. for a(ℓ,m)

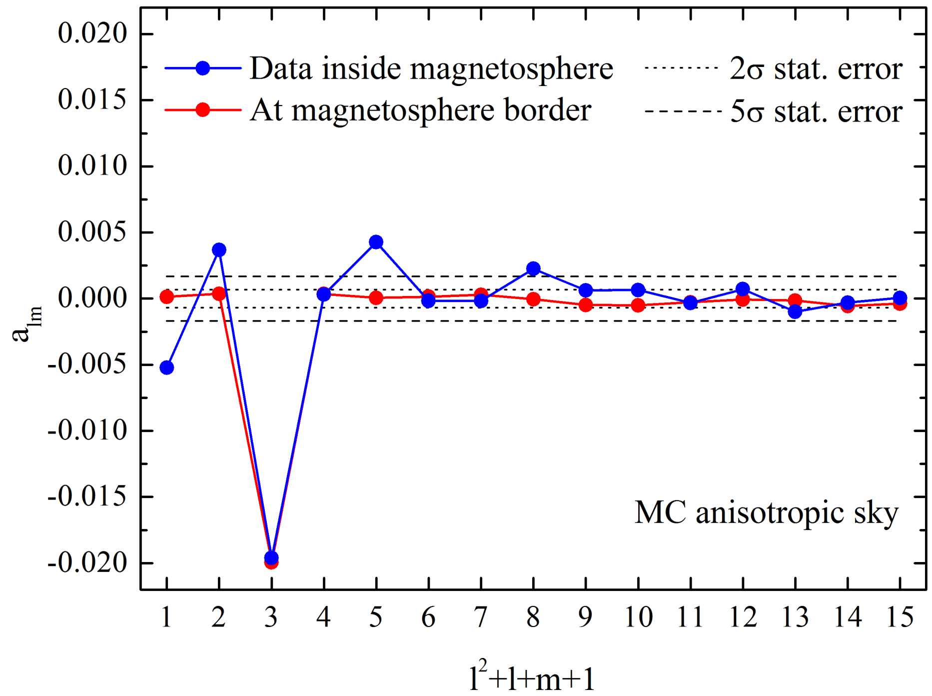
$a_{10}=0$

$a_{11}=0$

$a_{1-1}=-0.02$

- Outside magnetosphere only one coefficient is different from zero, as expected.

...and for the sky inside magnetosphere



AT MAGNETOSPHERE BORDERS
and
INSIDE MAGNETOSPHERE

Simulated signal:

$\rho_{NS} = 0$

$\rho_{BF} = 0$

$\rho_{EW} = -0.01$

i.e. for a(ℓ,m)

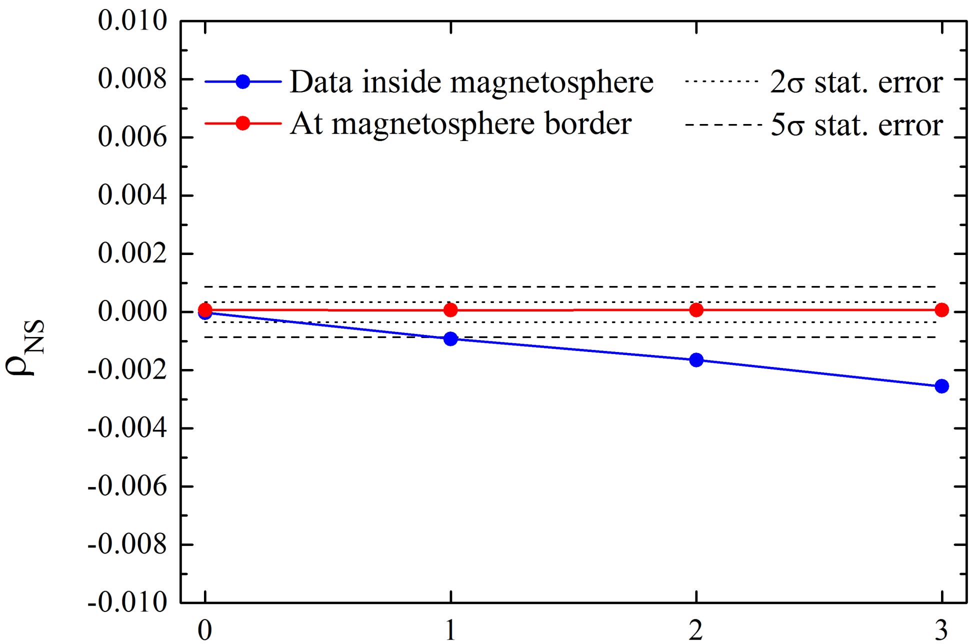
$a_{10} = 0$

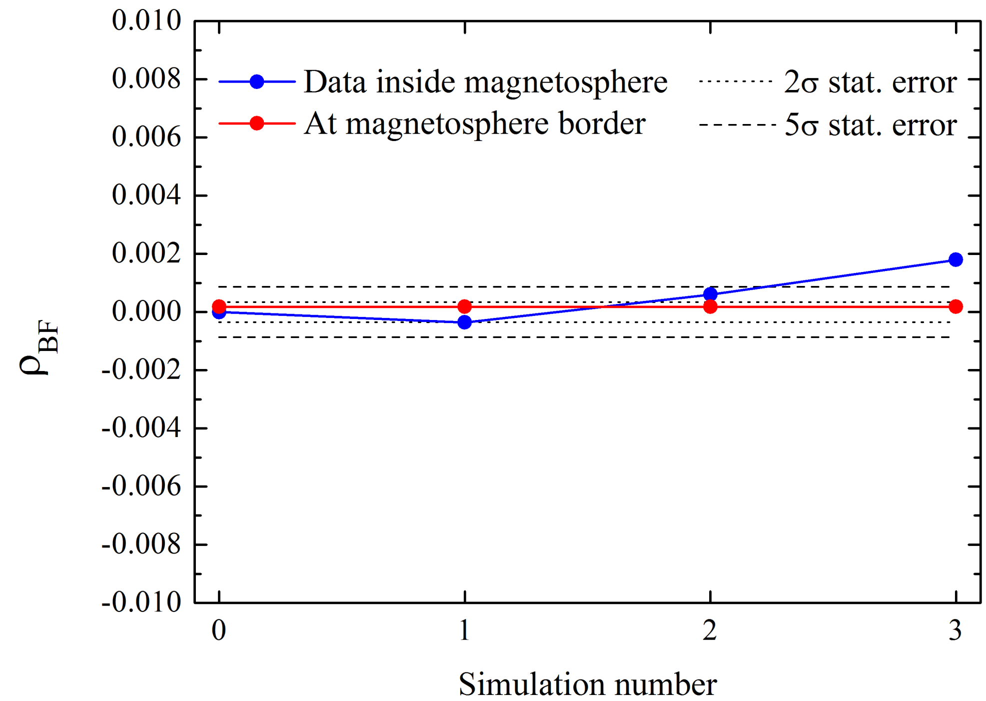
$a_{11} = 0$

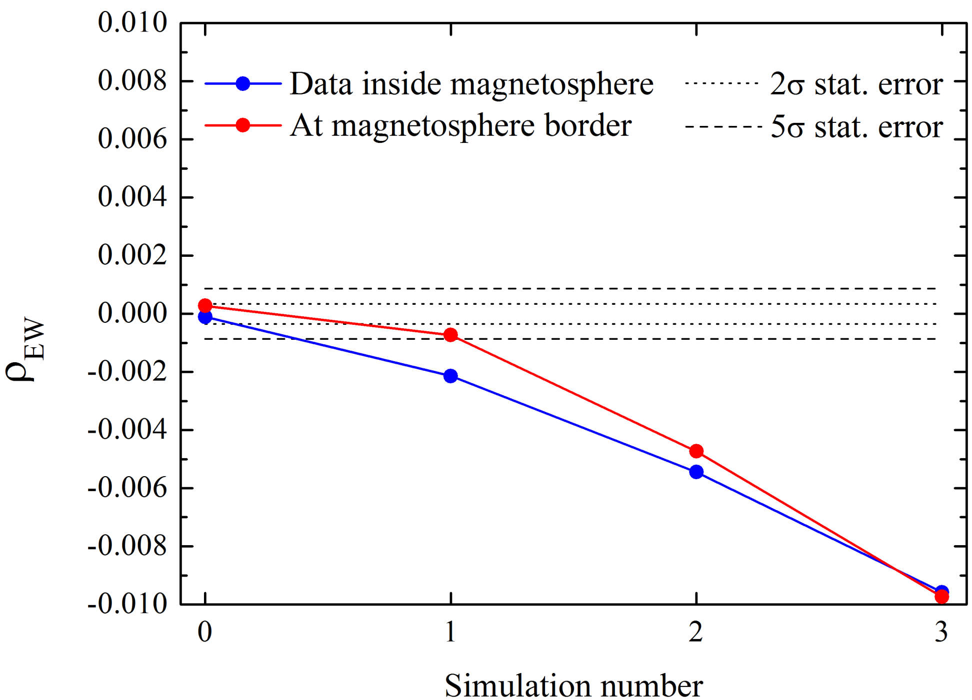
$a_{1-1} = -0.02$

- Outside magnetosphere only one coefficient is different from zero, as expected.
- Magnetosphere introduces a **rotation of the true sky**.
- The EW signal inside the magnetosphere is spread over other multi-polar components.

Various simulated anisotropic skies





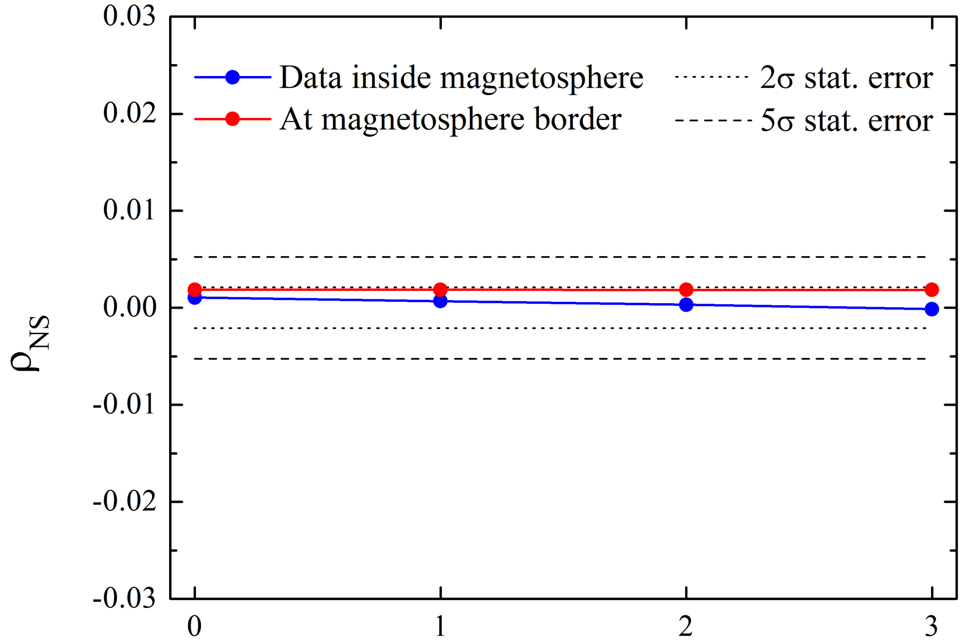


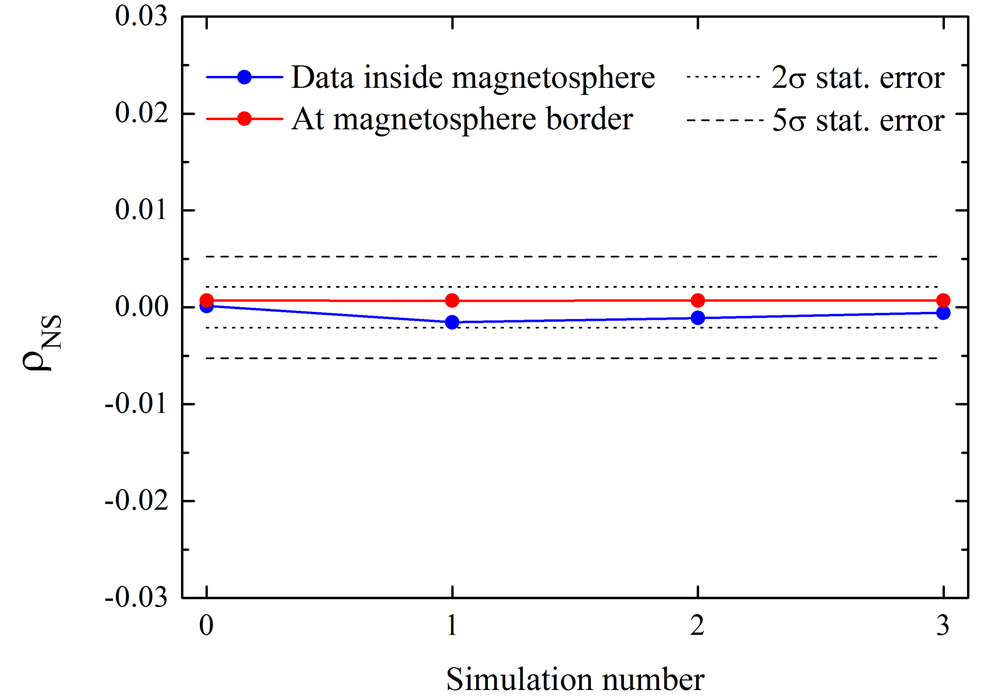
Simulated signals (at magn. border):

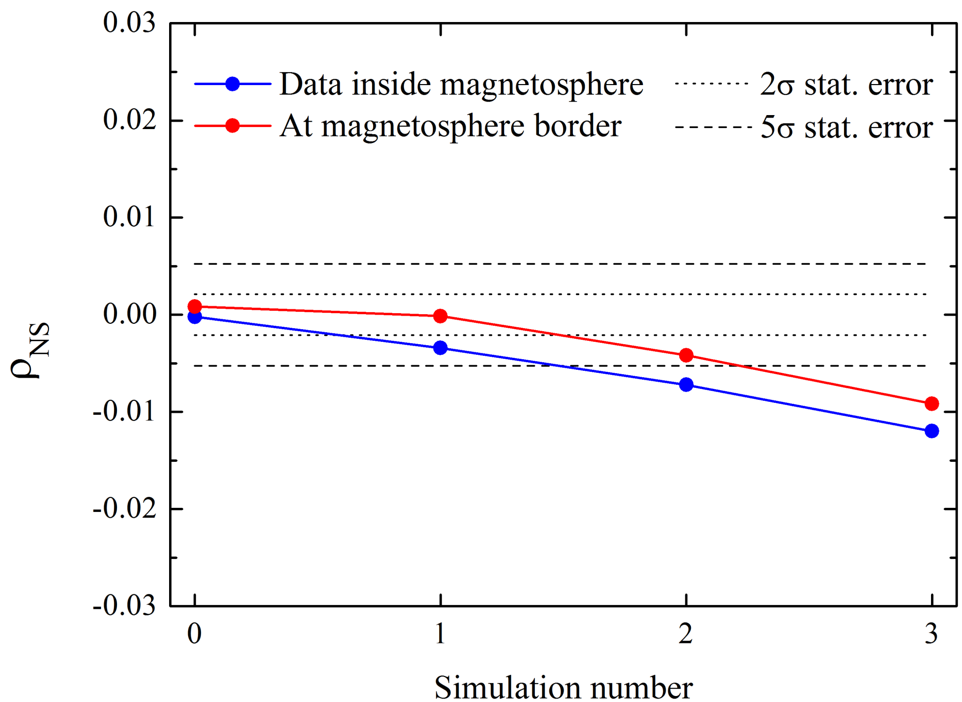
Sim. Num.	0	1	2	3
ρ_{NS}	0	0	0	0
ρ_{BF}	0	0	0	0
ρ_{EW}	0	-0.001	-0.005	-0.01

In [30:100]GV

Reducing to high rigidity particles







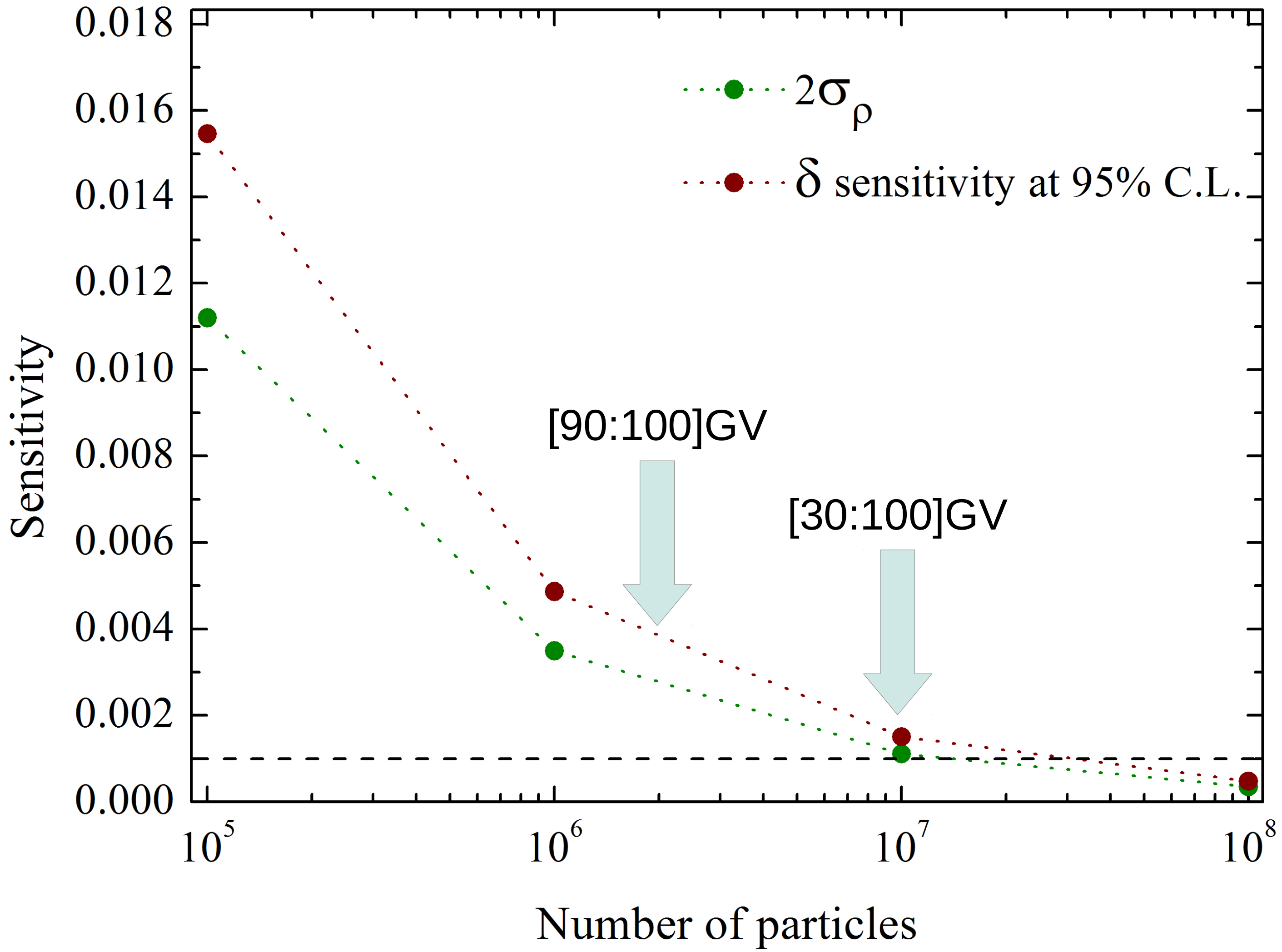
Simulated signals (at magn. border):

Sim. Num.	0	1	2	3
ρ_{NS}	0	0	0	0
ρ_{BF}	0	0	0	0
ρ_{EW}	0	-0.001	-0.005	-0.01

In [90:100]GV

Forecast of dipole sensitivity

Using extrapolated statistical limit.

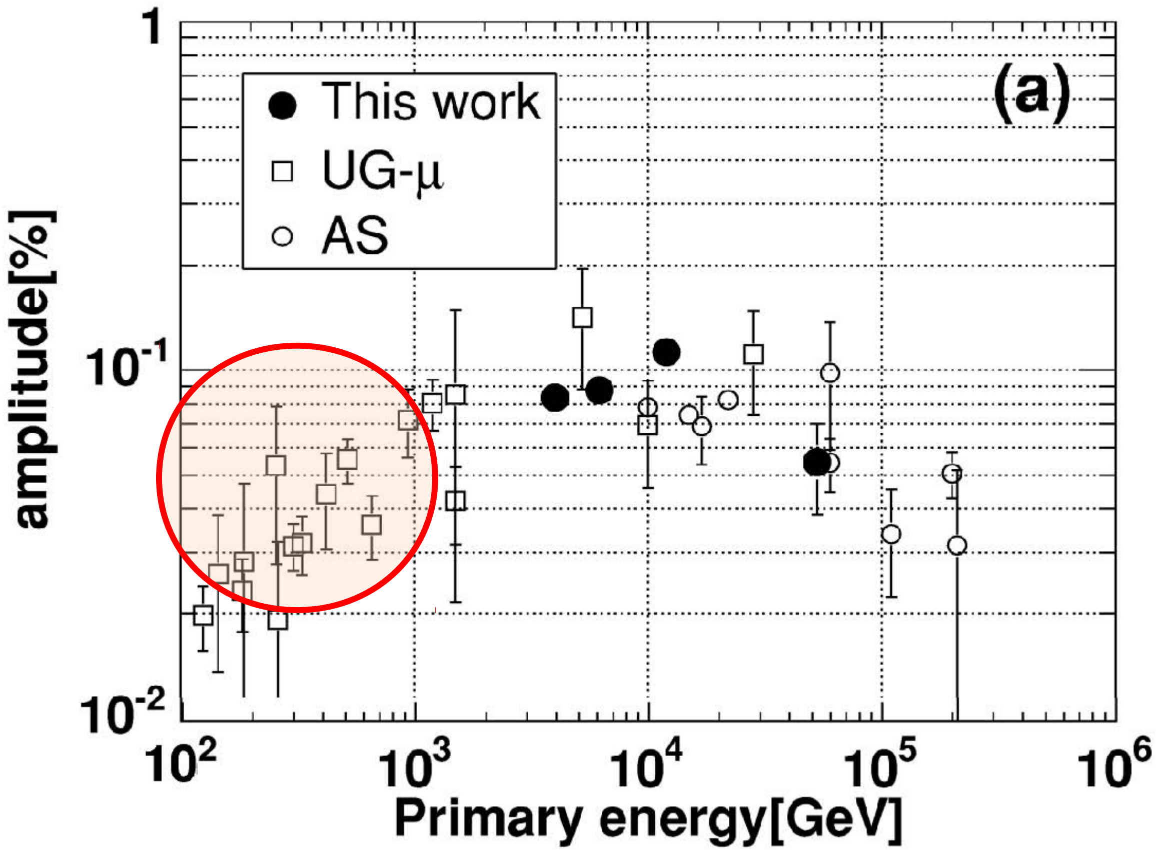


Conclusions

- We developed a method to determine particle deviations with respect to the expected isotropic sky distribution, through a MC approach;
- proton sky isotropic distribution and the corresponding monopole can be detected with an error ± 0.0001 which is in fully agreement with that expected from a pure isotropic distribution of 100M particles;
- proton sky reconstructed at magnetosphere borders is isotropic, as already predicted by Fermi and Rossi in 1933;
- the anisotropic signal detected inside magnetosphere is rotate with respect to the signal at the borders of the magnetosphere, due to the effect of the Earth magnetic field. The rotation is almost absent at higher rigidities;
- the present Monte Carlo method can be applied to any particle.

BACK-UP SLIDES

Proton isotropy in literature



From ground-based UHE observations: CR (**protons**)

dipole ≈ 2-6 · 10-4

between 100-500 GeV

and is lower below 100 GeV

See:

Amenomori et al. (TIBET AIR SHOWER ARRAY), *ApJ* **626**, L29–L32 (2005)

For a review:

Erlykina and Wolfendale, JCAP04 (2013) 006

Fermi E. and B. Rossi, Rend. R. Accad. Naz. Lincei **17**, 346 (1933)

Fisica. — *Azione sul campo magnetico terrestre sulla radiazione penetrante.* Nota[1] del Corrisp. E. FERMI e B. ROSSI.

D'altra parte si dimostra facilmente che se consideriamo uno sciame di elettroni (o protoni) aventi tutti la stessa energia, che si muovono in un campo magnetico qualsiasi, e seguiamo nel loro movimento quelli che all'istante iniziale si trovano in un dato elemento di volume e hanno velocità appartenenti a un dato elemento di angolo solido, il prodotto dell'elemento di volume nel quale essi sono contenuti, per l'elemento di angolo solido a cui appartengono le loro velocità resta costante durante il movimento dello sciame di corpuscoli. Da questa proprietà, analoga al teorema di Liouville, e dall'ipotesi che la distribuzione degli elettròni a distanza infinita dalla Terra sia uniforme ed isotropa segue immediatamente che l'intensità specifica della radiazione penetrante (numero di corpuscoli, per intervallo unitario di energia e per angolo solido unitario, che attraversano normalmente una superficie unitaria nell'unità di tempo) è per ogni punto (prescindendo dall'assorbimento atmosferico) la stessa in tutte le direzioni, da cui i corpuscoli possono effettivamente giungere.

Theory of the Geomagnetic Effects of Cosmic Radiation.

By

MANUEL SANDOVAL VALLARTA.

There is a fundamental property of all allowed directions to which we wish to call attention at the outset. From LIOUVILLE's theorem on the conservation of volume element in phase space it follows that the intensity of cosmic particles in any allowed direction, defined as the number of particles of given energy crossing unit solid angle per unit time, is the same as it is at their starting point. Therefore, if the distribution at infinity is isotropic, the intensity is the same in all allowed directions for any given energy. This important feature of the dynamical problem, which FERMI and ROSSI and independently LEMAÎTRE and the author pointed out [5] in 1932, results in a major simplification in the physically important problem of calculating intensities. If the distribution at large distances from the earth is isotropic, then it is only necessary to find the allowed cone and multiply the subtended solid angle by the intensity in any allowed direction, for any given energy.

Proton isotropic background

M.Vallarta, Encyclopedia of Physics Volume 9 / 46 / 1, 1961, pp 88-129

Theory of the Geomagnetic Effects of Cosmic Radiation.

By

MANUEL SANDOVAL VALLARTA.

If all directions are allowed, an isotropic distribution of particles keeps its isotropy when traveling through the magnetosphere. This property is charge independent.

See also Fermi E. and B. Rossi, Rend. R. Accad. Naz. Lincei **17**, 346 (1933)