Solar Particle Energy Spectra by Application of Regularization Methods

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International Space Science Institute, 24 March 2006

Outline

- SOHO spacecraft
- EPHIN sensor
- Regularization
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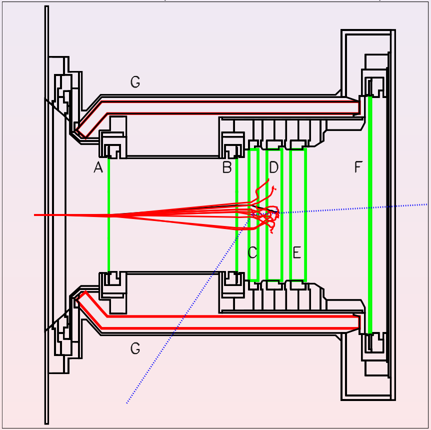
- EPHIN kernel
- Solar particle events and spectra
- Conclusion

The EPHIN in the GEANT 4 simulation

EPHIN Sensor

One of the experiments on SOHO is the sensor **EPHIN**, devoted to measure solar energetic particles (**e**lectron, **p**roton, **h**elium **in**strument)



Side view of the sensor, a tube, consisting of active detectors, 6 Si-solid state detectors **A, B, C, D, E, F** (in green) and an anticonincidence scintillator G (in red), providing information on energy loss. Included are 10 tracks of 5 MeV electrons (MC-simulation)

Measuring channels are defined with requirements on energy deposit and range of particles, four **E**-channels, mainly detecting electrons, and four **P**-channels, mainly detecting protons.

Regularization

The Eq.(1) is the Fredholm integral equation of the first kind and after the discretization one must solve the singular or ill-conditioned system of linear algebraic equations

$$\mathbf{A}f = z, \tag{1}$$

where $\mathbf{A} \in R^{m\times n}$ is a matrix with elements a_{ij}, $f \in R^n$ and $z \in R^m$ are vectors. Using the singular value decomposition (SVD) method we have the minimum norm solution of least squares problem

$$f_{svd} = \mathbf{A}^{+}z = \mathbf{V}\mathbf{\Sigma}^{+}\mathbf{U}^{*}z. \tag{2}$$

The iterated Tikhonov regularization method

$$f_{k+1} = f_k + (\mathbf{A}^{*}\mathbf{A} + \alpha\mathbf{I})^{-1}\mathbf{A}^{*}r_k \qquad \text{with} \qquad r_k = z - \mathbf{A}f_k \tag{3}$$

$f_0 = 0$ gives the solution f_{tikh}, that is completely congruent with the solution f_{svd}. The nonnegative solution f_{opt} we obtain by the optimization with constrains

$$y(f) = \min \sum_{i=1}^{m} \left(\frac{z_i - \sum_{j=1}^{n} a_{ij} f_j}{s_i}\right)^2 \qquad \text{subject to} f_i \geq 0 (i = 1, ..., n). \tag{4}$$

and with $s_i^2 = z_i$. f_{tikh} or f_{svd} is used as the starting value for this minimization problem.

Integral Equation

To derive the energy spectrum $I(E_{beam})$ from experimental data $I(E_{deposit})$ the integral equation

$$I(E_{deposit}) = \int \mathbf{K}(E_{deposit}, E_{beam}) \cdot I(E_{beam}) \cdot dE_{beam} \tag{1}$$

has to be solved. The kernel **K** can be derived from a Monte Carlo simulation of the sensor response

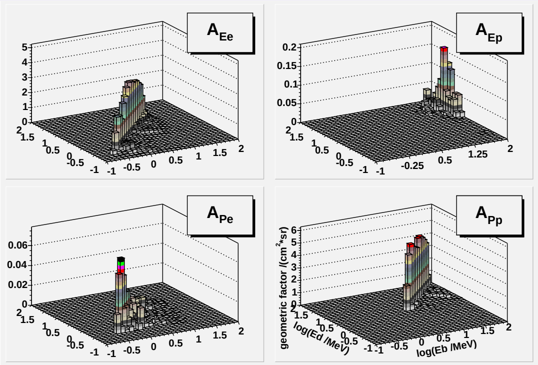
$$I(E_{deposit}) \cdot dE_{deposit} = \int (dF \cdot d\Omega \cdot dE_{beam} \cdot \epsilon(x, y, \theta, \phi, E_{beam}, E_{deposit})) \cdot dE_{deposit} \tag{2}$$

The kernel **K** is

$$\mathbf{K} = \int dF \cdot d\Omega \cdot \epsilon(x, y, \theta, \phi, E_{beam}, E_{deposit}) \tag{3}$$

where ϵ is the efficiency to detect a solar particle with energy E_{beam} within the interval dE_{beam}, generating the energy deposit $E_{deposit}$, within the interval $dE_{deposit}$, when arriving in the area element dF at position (x,y) and in the solid angle element $d\Omega$ from direction (θ, ϕ). The detection efficiency ϵ and **K** is calculated via the Monte Carlo technique, usually assuming an isotropic angular distribution of the beam, with a uniform distribution on the sensor area.

EPHIN kernel



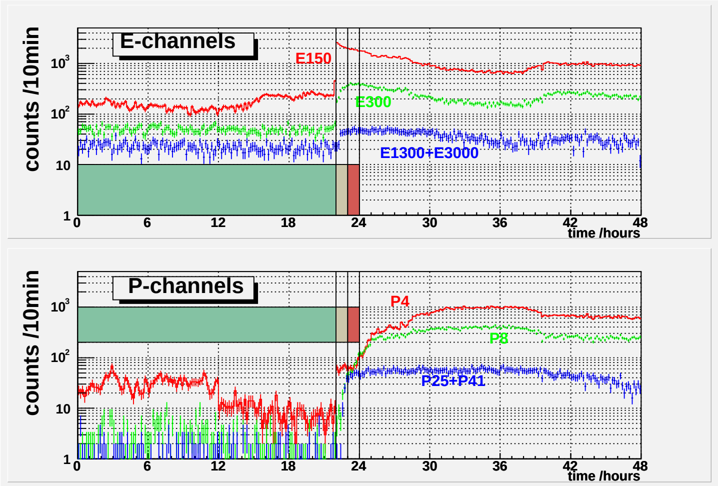
Geometric factor in dependence on E_{beam} and $E_{deposit}$ in the **E**-channels and the **P**-channels, from a Monte Carlo simulation of the EPHIN instrument

The Eq.(3) is considered in the following common form

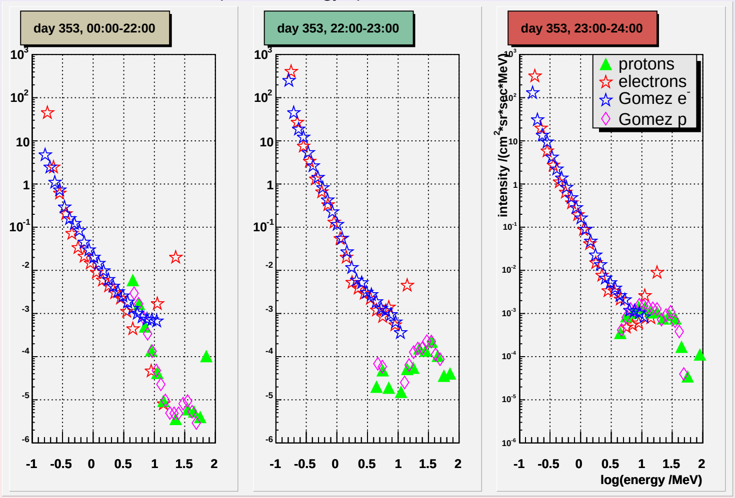
$$\begin{pmatrix} \mathbf{A_{E_e}} & \mathbf{A_{E_p}} \\ \mathbf{A_{P_e}} & \mathbf{A_{P_p}} \end{pmatrix} \begin{pmatrix} f^{(e)} \\ f^{(p)} \end{pmatrix} = \begin{pmatrix} z^{(E)} \\ z^{(P)} \end{pmatrix}$$

with the matrices plotted in the graphic, $f^{(p)}(E_{beam})$ and $f^{(e)}(E_{beam})$ describe the proton and the electron spectrum respectively, leading to the observed $z^{(P)}(E_{deposit})$ and $z^{(E)}(E_{deposit})$ distributions.

Solar energetic particle event, day 353-354 in 2002

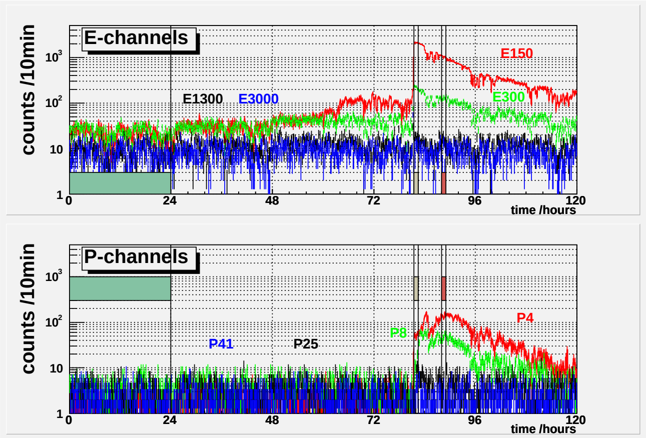


time propagation of countrate during solar energetic particle event, starting on day 353/2002 at about 22.30 in time in the E-channels, a bit later in the P-channels particle energy spectra at selected times

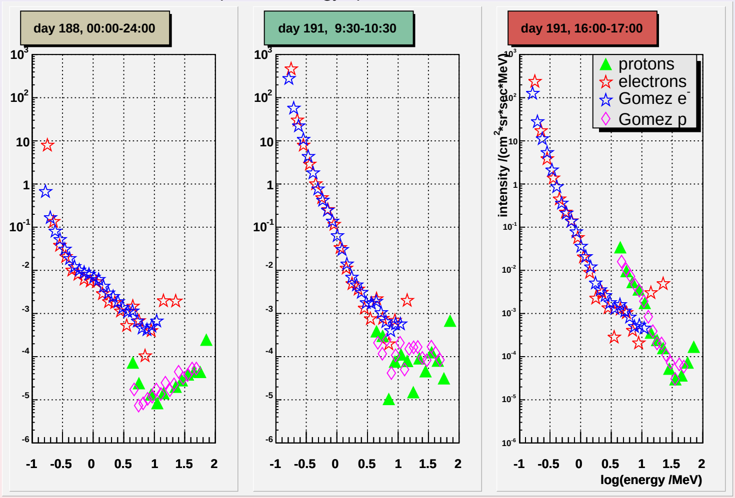


proton and electron flux on day 2002/353 from the present analysis together with results from Gomez obtained by another method

Solar energetic particle event, day 188-192 in 1996



time propagation of countrate during solar energetic particle event, starting on day 188-192 1996 at about 9.30 in time on day 191 in the E-channels, a bit later in the P-channels particle energy spectra at selected times



proton and electron flux on day 188-192 in 1996 from the present analysis together with results from Gomez obtained by another method

Conclusion

The analysis of experimental data is simplified and obtained in a more general way by solving the Inverse Problem.

No special a priori assumptions on the shape of distributions, as usually applied, and the contribution of various particle types into defined channels can be taken into account.